

Close Wed: HW\_5A, 5B, 5C (7.1,7.2,7.3)

Entry Tasks: Fill in the blanks

Square Identities
$\sin^2(x) =$
$\cos^2(x) =$
$\sec^2(x) =$
$\tan^2(x) =$

Half Angle Identities
$\sin^2(x) =$
$\cos^2(x) =$
$\sin(x)\cos(x) =$

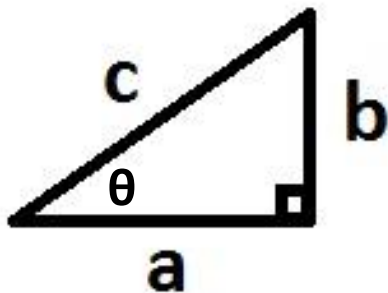
What are these in terms of a, b, and c?

$\sin(\theta) =$

$\cos(\theta) =$

$\tan(\theta) =$

$\sec(\theta) =$



What is the first step in each integral below?

$$\int \sin^3(x)\cos^4(x)dx$$

$$\int \sin^5(x)\cos^3(x)dx$$

$$\int \cos^4(x)dx$$

$$\int \tan^5(x)\sec^4(x)dx$$

$$\int \tan^5(x)\sec(x)dx$$

## 7.3 Trigonometric Substitution

*Goal:* Develop a method for evaluate integrals involving expressions of the form

$\sqrt{a^2 - x^2}$ ,  $\sqrt{a^2 + x^2}$ , or  $\sqrt{x^2 - a^2}$

### 3 Types of Problems/Motivation

1.  $a^2 - x^2$ , *example:*

$$\int \frac{x^3}{\sqrt{4 - x^2}} dx$$

2.  $a^2 + x^2$ , *example:*

$$\int x^2 \sqrt{9 + x^2} dx$$

3.  $x^2 - a^2$ , *example:*

$$\int \frac{\sqrt{x^2 - 16}}{x} dx$$

CASE	SUBSTITUTION
$a^2 - x^2$	$x = a \sin(\theta), \quad -\pi/2 \leq \theta \leq \pi/2$
$a^2 + x^2$	$x = a \tan(\theta), \quad -\pi/2 < \theta < \pi/2$
$x^2 - a^2$	$x = a \sec(\theta), \quad 0 \leq \theta < \pi/2, \text{ (pos. } x)$ $\pi \leq \theta < 3\pi/2 \text{ (neg. } x)$

Trigonometric Substitution Method:

- A) Make the substitution.  
Don't forget  $dx = ??? d\theta$ .  
Use trig identity (eliminate root)
- B) Use 7.2 methods for trig integrals.
- C) Draw a triangle and return to  $x$ .

Completing the Square:

$$\sqrt{ax^2 + bx + c}$$

If you ever encounter a “**middle term**” (like **bx** above), then you need to complete the square.

*Example:*  $\sqrt{64 - 24x - 4x^2}$

i) **Factor out the “a”.**

$$\sqrt{4(16 - 6x - x^2)} = 2\sqrt{16 - 6x - x^2}$$

ii) **Add and subtract**

**“half the middle squared”**

Half of middle =  $(-6)/2 = -3$

Squared =  $(-3)^2 = 9$

$$2\sqrt{16 + 9 - 9 - 6x - x^2}$$

iii) **Factor the perfect square**

$$2\sqrt{25 - (x + 3)^2}$$

iv) **Check your work!**